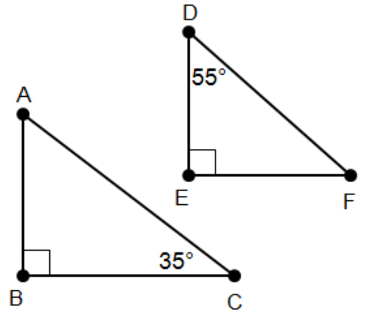
SM2 8.2: Similarity

Practice Exercises:

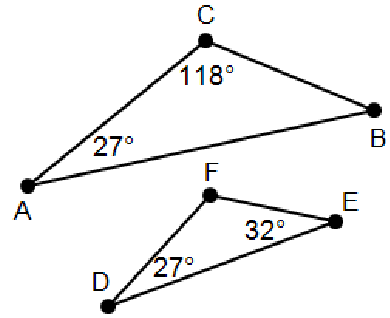
Determine if the following triangles are similar. If so, write a similarity statement.

1)



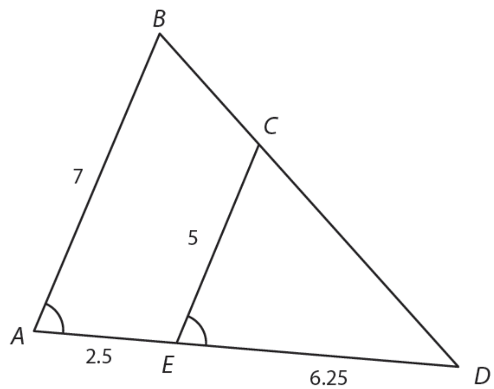
$\angle A = 180° - 90° - 35° = 55°$
$\angle F = 180° - 90° - 55° = 35°$
By AA Triangle Similarity $\triangle ABC \sim \triangle DEF$

2)



$\angle B = 180° - 27° - 118° = 35°$
$\angle F = 180° - 27° - 32° = 121°$
$\triangle ABC$ is not similar to $\triangle DEF$

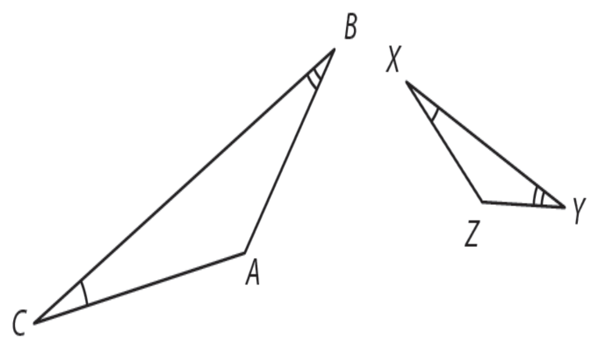
3)



$\frac{AB}{EC} = \frac{7}{5} = 1.4, \quad \frac{AD}{ED} = \frac{2.5 + 6.25}{6.25} = \frac{8.75}{6.25} = 1.4$

$\angle A \cong \angle E$
By SAS Triangle Similarity $\triangle ABD \sim \triangle ECD$
Or just use AA because it's a lot easier!

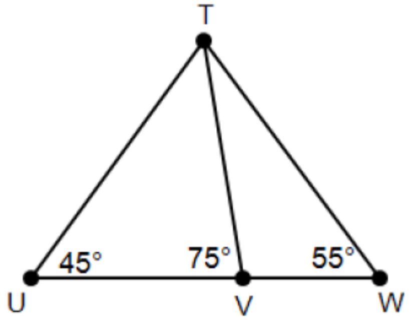
4)



$\angle C \cong \angle X$
$\angle B \cong \angle Y$

By AA Triangle Similarity $\triangle ABC \sim \triangle ZYX$

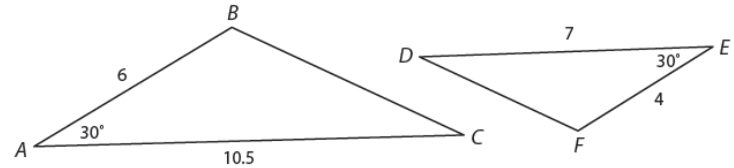
5)



$\angle TVW = 180° - 75° = 105°$
$\angle WTV = 180° - 55° - 105° = 20°$
$\angle UTV = 180° - 75° - 45° = 60°$

$\triangle WTV$ is not similar to $\triangle UVT$ and is not similar to $\triangle UTW$

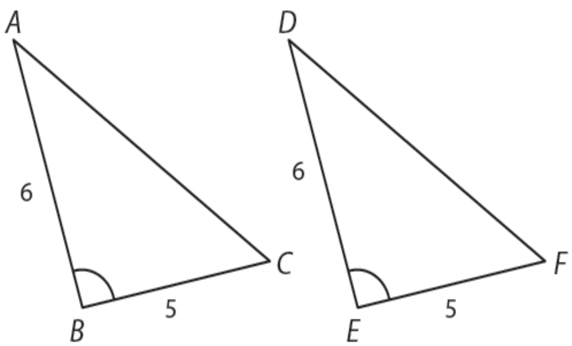
6)



$\frac{AB}{EF} = \frac{6}{4} = 1.5, \quad \frac{AC}{ED} = \frac{10.5}{7} = 1.5$

$\angle A \cong \angle E$
By SAS Triangle Similarity $\triangle ABC \sim \triangle EFD$

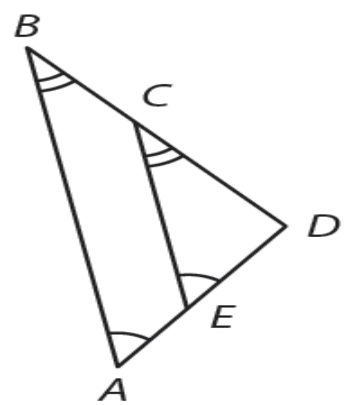
7)



$\frac{AB}{DE} = \frac{6}{6} = 1, \quad \frac{DE}{EF} = \frac{5}{5} = 1$

$\angle B \cong \angle E$
By SAS Triangle Similarity $\triangle ABC \sim \triangle DEF$
The triangles are also congruent.

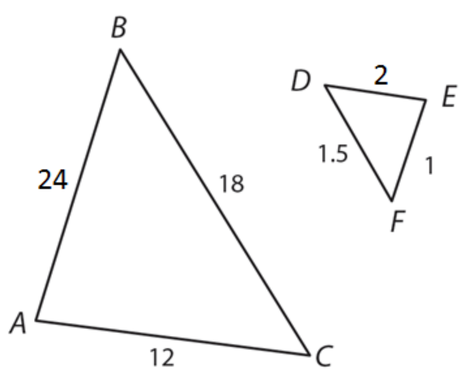
8)



$\angle B \cong \angle C$
$\angle A \cong \angle E$
$\angle D \cong \angle D$

By AA Triangle Similarity $\triangle ABD \sim \triangle ECD$

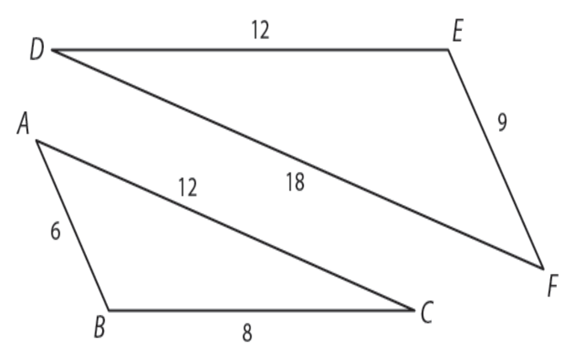
9)



$\frac{BC}{DF} = \frac{18}{1.5} = 12, \quad \frac{AB}{DE} = \frac{24}{2} = 12, \quad \frac{AC}{EF} = \frac{12}{1} = 12$

By SSS Triangle Similarity $\triangle ABC \sim \triangle EDF$

10)

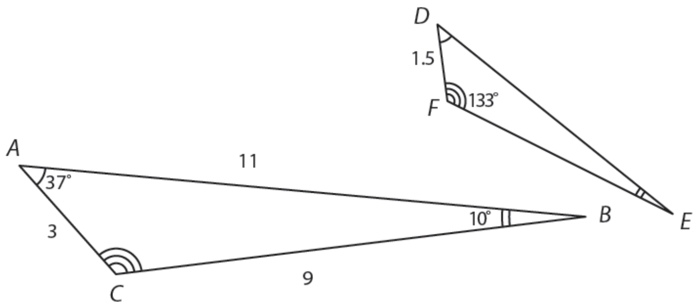


$\frac{BC}{DE} = \frac{8}{12} = .67, \quad \frac{AC}{DF} = \frac{12}{18} = .67, \quad \frac{AB}{EF} = \frac{6}{9} = .67$

By SSS Triangle Similarity $\triangle ABC \sim \triangle FED$

Find all the angle measures and side lengths for each triangle of the given similar pairs.

11) $\Delta ABC \sim \Delta DEF$



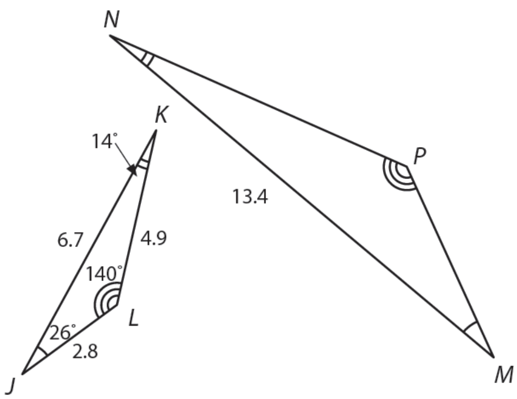
$\angle A \cong \angle D = 37°$
$\angle B \cong \angle E = 10°$
$\angle C \cong \angle F = 133°$

$$\frac{AC}{DF} = \frac{3}{1.5} = 2$$

$$Find\ EF: \frac{9}{EF} = 2$$
$$2EF = 9$$
$$EF = \frac{9}{2} = 4.5$$

$$Find\ DE: \frac{11}{DE} = 2$$
$$2DE = 11$$
$$DE = \frac{11}{2} = 5.5$$

12) $\Delta JKL \sim \Delta MNP$



$\angle J \cong \angle M = 26°$
$\angle K \cong \angle N = 14°$
$\angle L \cong \angle P = 140°$

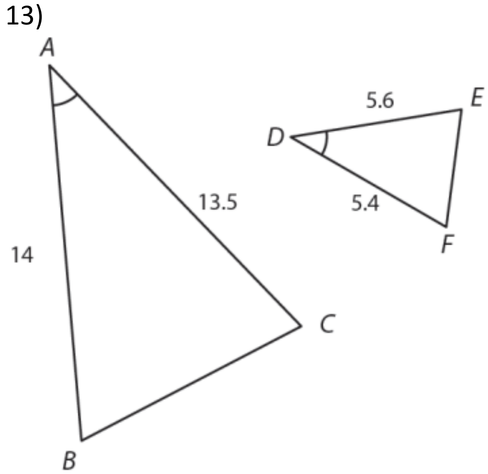
$$\frac{MN}{JK} = \frac{13.4}{6.7} = 2$$

$Find\ MP:$
$EF = 2(2.8)$
$MP = 5.6$

$Find\ NP: DE$
$= 2(4.9)$
$NP = 9.8$

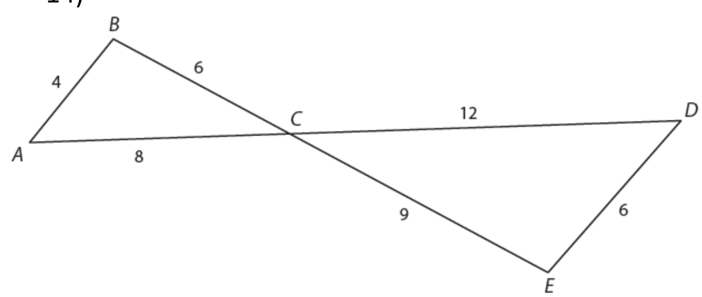
Prove that the triangles are similar.

13)



Proof,
We are given that $\angle A \cong \angle D$. We want to show that the proportions of the two corresponding sides of the two triangles are equal. $\frac{AB}{DE} = \frac{14}{5.6} = 2.5, \frac{AC}{DF} = \frac{13.5}{5.4} = 2.5$.
Hence $\frac{AB}{DE} = \frac{AC}{DF}$. Hence, by SAS Triangle Similarity $\Delta ABC \sim \Delta DEF$

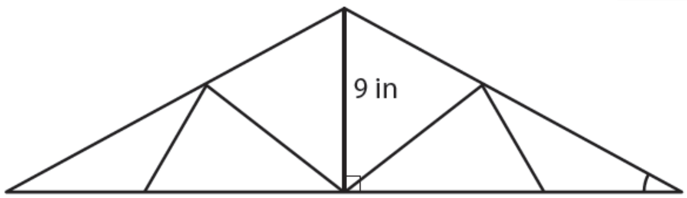
14)



Proof,
We are given the side lengths of the two triangles. We want to show that the proportions of three corresponding sides of the triangles are equal. $\frac{AB}{DE} = \frac{4}{6} = \frac{2}{3}, \frac{AC}{DC} = \frac{8}{12} = \frac{2}{3}, \frac{BC}{EC} = \frac{6}{9} = \frac{2}{3}$. So all the sides are proportional and $\Delta ABC \sim \Delta DEC$ by SSS Triangle Similarity

Application Problems:

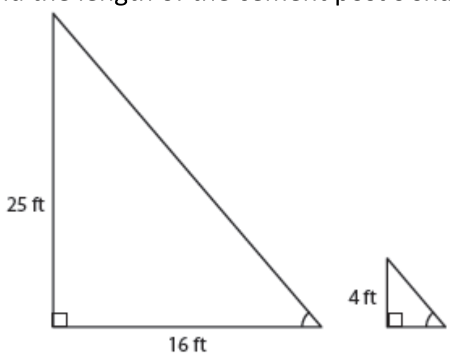
15) The support beams of truss bridges are triangles. James made a model of a truss bridge with a scale of 1 inch = 4 feet. If the height of the tallest triangle on the model is 9 inches, what is the height of the tallest triangle on the actual bridge?



Since the scale is 1 inch = 4 feet = 48 inches, the scale factor is 48 inches.

The height of the tallest triangle on the actual bridge is : $9 \times 48\ inches = 432\ inches\ or\ 36\ feet$

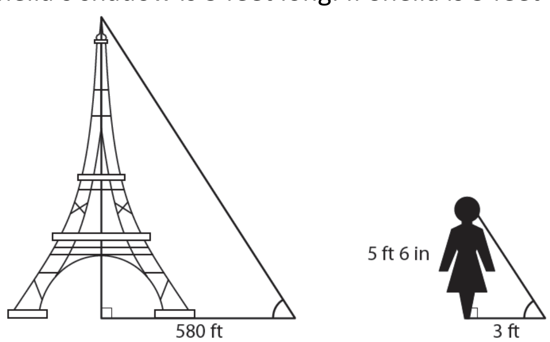
16) A statue that is 25 feet tall casts a shadow that is 16 feet long. A cement post next to the statue is 4 feet tall. Find the length of the cement post's shadow.



By AA triangle similarity the two triangles are similar. So I can use the proportions of the sides to find the length of the shadow. I will call the length x.

$$\frac{25}{4} = \frac{16}{x}$$
$$25x = 16 \times 4$$
$$25x = 64$$
$$x = \frac{64}{25} = 2.56\ feet$$

17) Sheila is standing near the Eiffel Tower in Paris, France. The shadow of the monument is 580 feet long, and Sheila's shadow is 3 feet long. If Sheila is 5 feet 6 inches tall, how tall is the monument?

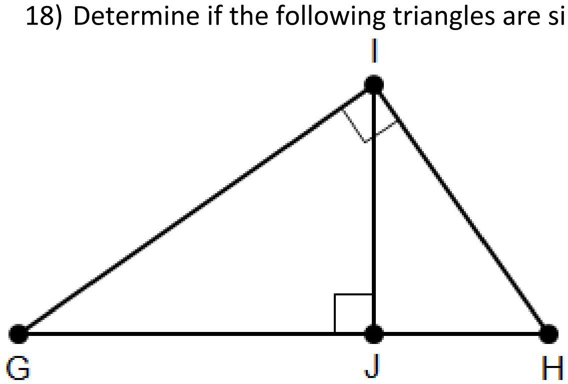


By AA triangle similarity the two triangles are similar. So I can use the proportions of the sides to find how tall the monument is. I will call the height x.
(Note that 5 ft 6 in is 5.5 feet)

$$\frac{580}{3} = \frac{x}{5.5}$$
$$3x = 580 \times 5.5$$
$$3x = 3190$$
$$x = \frac{3190}{3} = 1063.33\ ft\ or\ 1063\ ft\ 4\ in$$

Challenge Problem:

18) Determine if the following triangles are similar. Note that there are three triangles in the diagram.



We are given that, $\angle \boldsymbol{IJG} \cong \angle \boldsymbol{IJH} \cong \angle GIH$ (all 90°). And since $\angle H \cong \angle H$ by the reflexive property, then $\Delta HIJ \sim \Delta GHI$ (little ~ big). And since $\angle G \cong \angle G$ by the reflexive property, then $\Delta GIJ \sim \Delta GHI$ (mid ~ big).

Therefore, $\Delta HIJ \sim \Delta GHI \sim \Delta GIJ$ (little ~ big ~ mid) by the transitive property.